

# 1 Introduction

Octave provides functions to plot the frequency response of LTI systems. Examples of such functions are `bode`, `nyquist` and `nichols`. These functions take the frequency points at which the system response is to be calculated as an argument. The argument is optional. In calls where a vector of frequency points are not given to the function, it chooses a set of frequency values itself. Currently, Octave simply selects 500 points which are logarithmically spaced from the minimum frequency to the maximum. Additionally, Octave inserts the natural frequencies of the poles and zeroes of the supplied system also to the above vector.

For systems with poles or zeros which have very small damping ratios, the logarithmically spaced points are not generally close enough to fully capture the rapid changes in the magnitude and phase in the frequency response.

A modification to the selection of the default frequency points is given in this report.

## 2 Methods

Two methods can be used for generating sufficiently close by points. First method is to append additional points close to the frequency of the poles and zeros apart from the log spaced default points. This method is similar to existing Octave technique. Instead of inserting only the frequency of the poles and zeros, some additional points can also be inserted at cose by frequencies. A thumb rule for frequency selection is given in next section.

Second method is to use very large number of points in the initial vector (say, 50,000 instead of 500 logspaced points) and then delete all points whose neighbours are close in magnitude as well as phase within some pre decided tolerance. But this increases the execution time significantly and still won't guarantee closely spaced points near peaks and notches.

## 3 Phase response of system with one pair of complex poles

$$\begin{aligned}
 F(s) &= \frac{1}{(s-p)(s-p^*)} \\
 &= \frac{1}{s^2 - 2(p+p^*)s + p \cdot p^*} \\
 &= \frac{1}{s^2 - 2\text{Re}(p)s + |p|^2} \\
 \angle(F(i\omega)) &= -\tan^{-1}\left(\frac{-2\text{Re}(p)\omega}{|p|^2 - \omega^2}\right) \\
 &\triangleq A
 \end{aligned}$$

Assuming that we require at least one frequency point for every  $x$  deg change of phase to correctly capture the shape of the frequency response, we can solve the above equation for  $\omega$ .

$$\begin{aligned}
 \tan(-A)(|p|^2 - \omega^2) &= -2\text{Re}(p)\omega \\
 0 &= \omega^2 + \frac{2\text{Re}(p)}{\tan(A)}\omega - |p|^2 \\
 \omega &= \frac{1}{2} \left( -\frac{2\text{Re}(p)}{\tan(A)} + \sqrt{\left(\frac{2\text{Re}(p)}{\tan(A)}\right)^2 + 4|p|^2} \right)
 \end{aligned}$$

$A$  can be chosen to vary from 0 to  $-180$  for a complex pole.

## 4 Magnitude response of system with one pair of complex poles

$$\begin{aligned}
 F(s) &= \frac{1}{(s-p)(s-p^*)} \\
 &= \frac{1}{s^2 - 2(p+p^*)s + p \cdot p^*} \\
 &= \frac{1}{s^2 - 2\text{Re}(p)s + |p|^2} \\
 |F(i\omega)| &\triangleq Y \\
 \frac{1}{Y^2} &= (|p|^2 - \omega^2)^2 + (2\text{Re}(p)\omega)^2 \\
 v &\triangleq \omega^2 \\
 \frac{1}{Y^2} &= |p|^4 + v^2 - 2|p|^2v + (2\text{Re}(p))^2v \\
 0 &= v^2 + ((2\text{Re}(p))^2 - 2|p|^2)v + |p|^4 - \frac{1}{Y^2} \\
 v = \omega^2 &= \frac{1}{2} \left( -((2\text{Re}(p))^2 - 2|p|^2) \pm \sqrt{((2\text{Re}(p))^2 - 2|p|^2)^2 - 4(|p|^4 - \frac{1}{Y^2})} \right)
 \end{aligned}$$

Unlike phase which varies through 180 degrees, there is no fixed magnitude variation. It can be guessed taking  $Y$  as log spaced points between the magnitude at 0 and  $|p|$  frequencies.

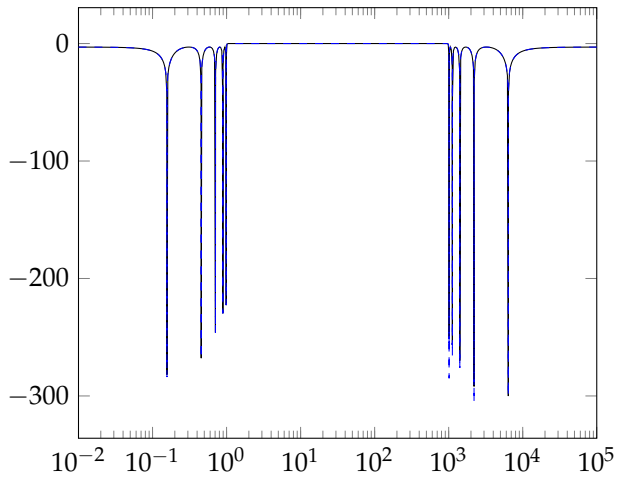
$$\begin{aligned}
 |Y|_{s=0} &= \frac{1}{|p|^2} \\
 |Y|_{s=j \cdot |p|} &= \frac{1}{2 \cdot \text{Re}(p) \cdot |p|}
 \end{aligned}$$

Since division is involved, and square root is taken twice, adequate protection should be provided to avoid Inf and NaN.

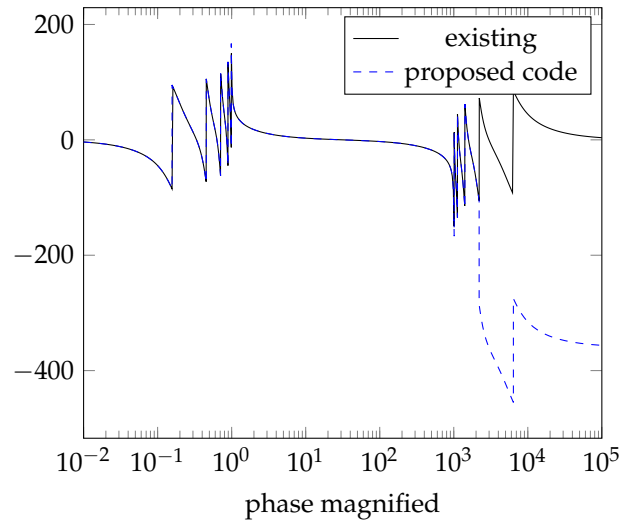
## 5 Results - Octave versus Modified code

Example uses a cheby2 filter. The results are not dramatic in the bode plots unless zoomed in. The results are slightly clearer in the Nyquist plot. Due to the hard coded magic numbers in currently submitted patch, the proposed code uses 1224 frequency points while existing code uses only 520 points (in the bode as well as Nyquist).

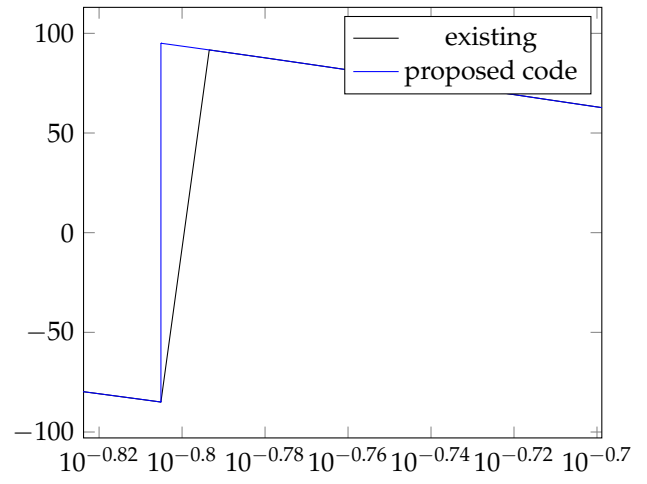
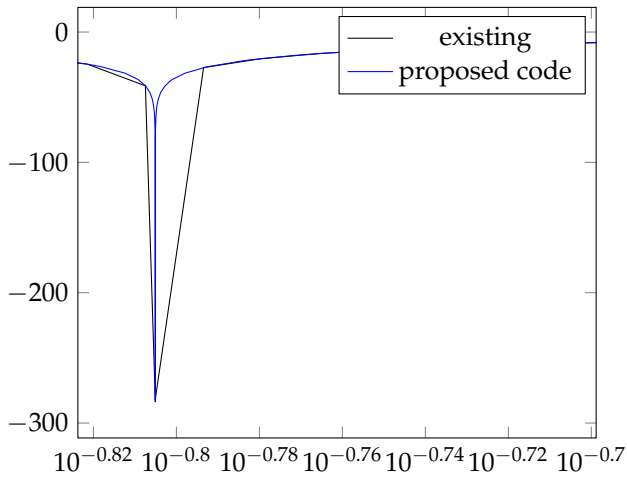
magnitude fullrange



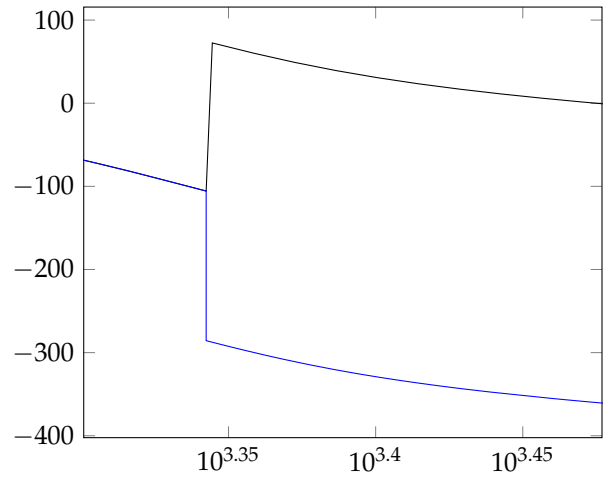
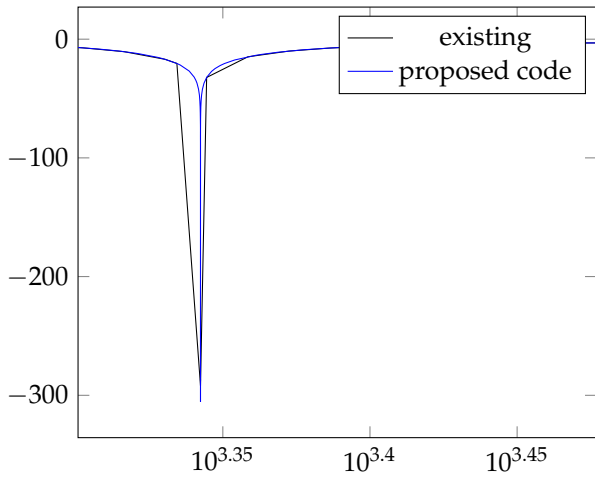
phase fullrange



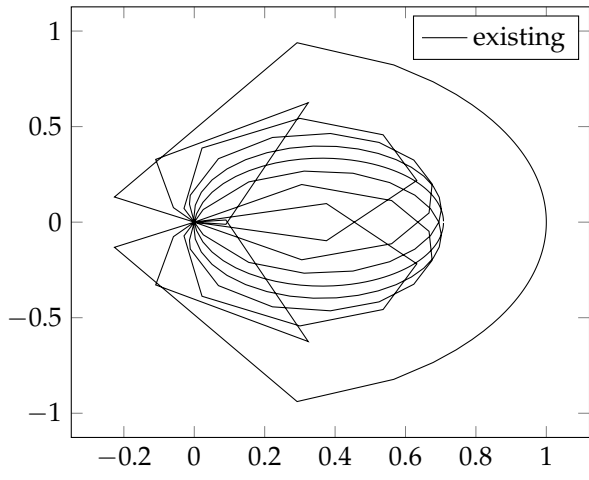
magnitude magnified



magnitude magnified



Nyquist



Nyquist

